

Floating phase in a dissipative Josephson junction array

Sumanta Tewari and John Toner

Department of Physics and Institute of Theoretical Science, University of Oregon, Eugene, OR 97403

Sudip Chakravarty

Department of Physics and Astronomy, University of California Los Angeles, Los Angeles, CA 90095-1547

(Dated: February 2, 2008)

We consider dissipative quantum phase transitions in Josephson junction arrays and show that the disordered phase in this extended system can be viewed as an unusual floating phase in which the states of local $(0+1)$ -dimensional elements (single Josephson junctions) can slide past each other despite arbitrary range spatial couplings among them. The unusual character of the metal-superconductor quantum critical point can be tested by measurements of the current voltage characteristic. This may be the simplest and most natural example of a floating phase.

PACS numbers:

It has recently been recognized [1] that phases and phase transitions in a given dimension can be embedded in a higher dimensional manifold. One realization of this is in a stack of two-dimensional (2D) layers of coupled XY spins with interlayer couplings along the layer-normals [1]. For a suitably chosen (and fairly complicated) set of competing interlayer couplings involving higher order gradients, it was shown in Ref. [1] that the 3D-system can exhibit 2D-behavior: power-law ordered phases in each layer below a characteristic temperature, T_{KT} , and a high-temperature disordered phase with exponentially decaying correlations. The transition separating the two phases remains 2D Kosterlitz-Thouless type. Below T_{KT} , the 2D power-law phases in each layer ‘slide’ past one another (a “floating” phase) [1]. Strikingly, such a phase is impossible for more conventional Josephson-type couplings between the layers. Another interesting case is the recently discussed quantum theory of the smectic metal state in stripe phases [2], which also involves gradient couplings. Unfortunately, there are no definitive experimental realizations of a floating phase, although there is some evidence for it in lipid-DNA complexes [3] in which lipid bilayers are able to slide over each other without cost in energy [4].

In this Letter we show that such a floating phase naturally occurs in the ground state of a system of coupled resistively shunted Josephson junctions [5]. The floating phase in this case is a state where the ground states of $(0+1)$ -dimensional elements (single Josephson junctions) can slide past each other despite coupling between them. Experimentally, this phase will behave like a metal with finite resistance[6, 7]. The striking point is that no additional, higher order gradient couplings need be invoked, in contrast to the stack of XY spins treated in Ref. [1].

The existence of such a floating phase in this system has been conjectured before[6, 8, 9]. The arguments presented there are, however, incomplete, since further than nearest-neighbor interactions, which are always present in any real system, are not considered in these treat-

ments. This is particularly worrisome since the treatment of XY stacks[1] found many regions of the phase diagram in which such interactions destroyed the floating phase. In this paper, we show that no such problem arises here: at least for sufficiently small further neighbor interactions, the floating phase is robust.

A concrete realization of longer ranged Josephson couplings relevant to the present model is a system of superconducting grains embedded in a metallic medium. The Josephson coupling J_{ij} between the grains at r_i and r_j falls off very slowly at $T = 0$, specifically $J_{ij} \propto \frac{1}{|r_i - r_j|^2}$ in two dimensions [10]; this is due to pair field-pair field correlation function in a normal metal.

The prior work also argued[6, 8, 9] that there will be a phase transition as the shunt resistance is varied, between the floating phase and a $(D+1)$ -dimensional superconducting phase, in which the phases of the junctions are coherently coupled. In addition to confirming this, we show from a renormalization group analysis that this transition, for suitable shunt resistance and small nearest neighbor Josephson coupling V , is governed by a line of quantum critical points with continuously varying critical exponents, which in dimensions higher than one is very rare indeed. As we discuss below, the upper critical dimension in this problem is infinity. The phase diagram is shown in Fig. 1. In this figure, $\alpha \equiv R/R_Q$, where R is the shunt resistance, and $R_Q \equiv h/4e^2$ is the quantum of resistance.

A similar phase diagram has been seen in a recent set of experiments [11], but the existence of continuously varying critical exponents is not yet established. We emphasize, however, that the floating phase is a metal, not an insulator: the metallic shunts short the current when the phase difference between the junctions become incoherent. The metallic behavior was not established in this experiment, but only the insulating behavior of the unshunted array. Clearly, further experiments at sufficiently low temperatures will be necessary in establishing the true nature of the floating phase. We expect that

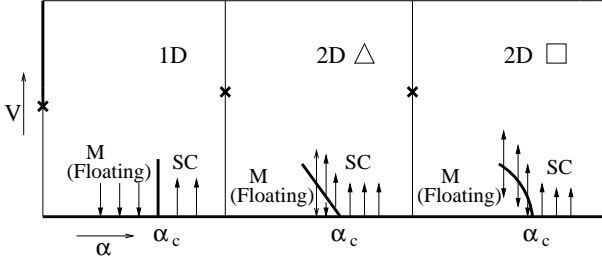


FIG. 1: Phase diagram and perturbative RG flows for 1D, 2D triangular lattice (3D face centered cubic), and 2D square lattice (3D simple cubic). M and SC indicate metal and superconductor, respectively. The thick lines correspond to line of fixed points. Crosses indicate special points on the $\alpha = 0$ axis denoting $(D + 1)$ -dimensional XY transition.

the current-voltage characteristic in the floating phase to be a non-universal power law controlled by dissipation α , similar to the results obtained previously [12]. In the superconducting state, the temperature-dependent non-universal power law will be similar to the classical XY model due to vortex unbinding in the presence of an imposed current [13].

As mentioned earlier, the phase transition is very strange, exhibiting continuously varying, non-universal critical exponents that depend on where the phase boundary is crossed. Additionally, the universality class of the transition depends *not at all* on the dimensionality of the system, but only on the local topology of the lattice. In particular, for a one-dimensional chain, the phase boundary in the V - α plane is vertical for small V (that is, independent of V), while for *any* lattice of *any* dimensionality that has closed loops of 3 nearest neighbor bonds (e.g., a 2D triangular or a 3D FCC lattice), the critical Josephson coupling V_c obeys, for α near α_c :

$$V_c(\alpha) = A(\alpha_c - \alpha), \quad (1)$$

where A is a non-universal constant. For *any* lattice of *any* dimensionality in which the smallest closed loops [14] have *more* than 3 nearest neighbor bonds (e.g., a 2D square or 3D simple cubic lattice), the critical Josephson coupling V_c obeys, for α near α_c :

$$V_c(\alpha) = \sqrt{A'(\alpha_c - \alpha)}, \quad (2)$$

where A' is another non-universal constant. We will hereafter refer to these distinct lattice types as “triangular-type” and “square-type” respectively.

One non-universal critical exponent that can easily be measured experimentally is the thermal exponent ν , which controls the scaling of characteristic temperatures T_{ch} with distance from the $T = 0$ quantum phase transition. One such characteristic temperature for α 's and V 's that lie in the superconducting phase at $T = 0$ is superconducting to normal transition temperature T_c . For α 's

and V 's that lie in the floating phase at $T = 0$, a characteristic temperature is the temperature at which the resistivity of the array as a whole has its minimum[7].

We predict that

$$T_{ch} \propto (\delta V)^\nu, \quad (3)$$

with $\delta V \equiv |V - V_c|$. With $\alpha_* \neq \alpha_c$ the value of α at which the transition occurs, the non-universal exponent $\nu(\alpha_*)$ is given by

$$\nu(\alpha_*) = \frac{1}{A''(\alpha_c - \alpha_*)}, \quad (4)$$

with $A'' = 1(2)$ for “triangular(square)-type” lattices.

The array of the Josephson-junctions coupled to local Ohmic dissipation is described by the following imaginary-time (τ) action:

$$\begin{aligned} \mathcal{S}/\hbar = & \int_0^\beta \left[\frac{1}{2E_0} \sum_i \left(\frac{\partial \theta_i}{\partial \tau} \right)^2 + V \sum_{\langle i,j \rangle} [1 - \cos \Delta \theta_{ij}(\tau)] \right] d\tau \\ & + \frac{\alpha}{4\pi} \sum_{\langle i,j \rangle} \frac{1}{\beta} \sum_n |\omega_n| |\Delta \tilde{\theta}_{ij}(\omega_n)|^2 + \mathcal{S}_J/\hbar \end{aligned} \quad (5)$$

Here the sum $\langle i,j \rangle$ is over nearest neighbor pairs in a lattice of arbitrary dimensionality and type. $\Delta \tilde{\theta}_{ij}(\omega_n)$ is a Fourier component of the phase difference between the two superconducting grains, $\Delta \theta_{ij}(\tau)$, with $\omega_n = \pm 2\pi n/\beta$: n is an integer, and β is the inverse-temperature. The charging energy of a grain, E_0 , is given by $E_0 \propto 1/C$, where C is the capacitance of the grain. The term containing α arises from integrating out the degrees of freedom of the harmonic-oscillator bath with a linear spectral-function (Ohmic dissipation) [15]. An Ohmic shunt, by definition, transfers charge continuously, while in quasiparticle tunneling in a superconductor the charge is transferred in discrete quantized amounts [16].

The term, \mathcal{S}_J , describes inter-junction couplings mediated by longer-range Josephson-interactions among the grains, which has not been considered previously. These couplings are described by the general action,

$$\mathcal{S}_J/\hbar = - \sum_{[s_i]} \sum_i \int_0^\beta d\tau J([s_i]) \cos \left[\sum_j s_j \theta(\vec{r}_i + \vec{\delta}_j, \tau) \right] \quad (6)$$

Here, $\theta(\vec{r}_i, \tau)$'s are the phases of the order parameters in the superconducting grains, the $\vec{\delta}_i$'s are arbitrary separation vectors on the lattice (to allow for arbitrary ranged couplings) and s_i is an integer-valued function of the layer number i satisfying $\sum_i s_i = 0$. This last constraint follows from the absence of an external field, which implies “rotation invariance” under adding the same constant to all of the θ 's. Note that the special case $s_0 = +1, \vec{\delta}_0 = \vec{0}, s_1 = -1, \vec{\delta}_1 = \vec{a}_\gamma$, where \vec{a}_γ is

a nearest neighbor vector, is just the nearest neighbor Josephson coupling V in Eq. (5).

To establish the existence of a dissipation-tuned quantum phase transition, we use a renormalization group analysis that is perturbative in V and J . We divide the field $\theta_i(\tau)$ into slow and fast components $\theta_{is}(\tau)$ and $\theta_{if}(\tau)$ and write the partition function Z as

$$Z = Z_0 \int \prod_i \mathcal{D}\theta_{is}(\tau) \exp \left[-\mathcal{S}_0^s/\hbar + \ln \langle e^{-\mathcal{S}'/\hbar} \rangle_{0f} \right] \quad (7)$$

Here Z_0 is a normalization constant, \mathcal{S}_0^s is the slow-frequency component of the quadratic part of the action containing the first and the third terms in Eq. (5), \mathcal{S}' contains the remaining two terms, and $\langle \dots \rangle_{0f}$ denotes the average with respect to the fast components of the quadratic part. The average in Eq. (7) contains a product of two factors involving V and J .

To leading order in V and J , the coupling constants do not mix. After computing the averages, we rescale τ , $\tau' = \tau/b$, b being the scale factor, to restore the original frequency cut-off, and then redefine the coupling constants to complete the renormalization. The dissipation term in Eq. (5) is dimensionless, and so it is held fixed by the RG. The first term then has τ -dimension -1 . Thus, it is irrelevant in the RG sense, and will henceforth be dropped for the remainder of this paper[17].

Writing $b = e^l$, where $l > 0$ is infinitesimal, and $z_0 = z/2$, where z is the coordination number of the lattice, we have to one loop order [8]:

$$\frac{dV}{dl} = \left(1 - \frac{1}{z_0\alpha}\right)V \quad (8)$$

In the absence of the J 's, the α -axis with $V = 0$ is a line of stable fixed points for $\alpha < 1/z_0$ and a line of unstable fixed points for $\alpha > 1/z_0$. The physical interpretation of the stable fixed points is as follows: for $\alpha < 1/z_0$, the barrier V among the different potential minima of the variable $\Delta\theta_{ij}$ is irrelevant, and the system described by $\Delta\theta_{ij}$ exhibits local quantum fluctuations.

This local criticality can be seen from the equal-time correlation function of the phase differences:

$$\langle \exp[iq\Delta\theta_{ij}(\tau)] \rangle \propto 1/(\beta\hbar\omega_c)^{\eta(q)}, \quad (9)$$

with $\eta(q) \equiv \min_{n \in \mathbb{Z}} \eta'(q-n)^2$, $\eta' \equiv 1/(z_0\alpha)$, q a real number, and $\hbar\omega_c = E_0$. Note that the simplest such correlation function one could imagine, namely $q = 1$, does *not* decay algebraically because $\eta(1) = 0$. This result for $\eta(q)$ is familiar for surface roughening [18, 19]. A similar calculation shows that the unequal time correlation functions of the bond variables are also critical:

$$\langle \exp[iq(\Delta\theta_{ij}(\tau) - \Delta\theta_{ij}(0))] \rangle = \frac{1}{(\omega_c\tau)^{2\eta(q)}}. \quad (10)$$

Although the correlation functions of the bond variables algebraically decay with time in any dimension, the same

is not true for the correlation of the site variables. We have computed the unequal time site correlation functions [20] and have found that they vanish as $\tau \rightarrow \infty$; the power law form is obtained for only $D \geq 3$. The locality of this phase follows from the result that [20], at zero temperature, spatial correlation functions of the grains, $\langle \exp[iq(\theta_i(\tau) - \theta_{i+r}(\tau))] \rangle$, are identically zero in any dimension; spatially separated grains are completely uncorrelated. Each junction is a dynamical system on its own, and there are only short-ranged correlations between spatially separated junctions.

For $\alpha > 1/z_0$, on the other hand, the barrier V grows, and the individual quantum systems saturate in one of the potential minima. V is like a local field term for the ‘bond-spin’ variable which becomes relevant above $\alpha = 1/z_0$, and orders the individual bond-spins. The transition at $\alpha = 1/z_0$ is purely local.

What happens when longer-ranged spatial couplings among them are introduced? Calculations precisely analogous to that leading to Eq. (8) give, to first order in J ,

$$\frac{dJ}{dl} = \left[1 - \frac{\Gamma([s_i])}{\alpha}\right]J, \quad (11)$$

with

$$\Gamma([s_i]) \equiv \sum_{i,j} s_i s_j U(\vec{\delta}_i - \vec{\delta}_j). \quad (12)$$

The ‘potential’ $U(\vec{r}) \equiv \frac{1}{N} \sum_{\vec{q}} (e^{i\vec{q}\cdot\vec{r}} - 1)/f_{\vec{q}}$, with $f_{\vec{q}} \equiv \sum_{\gamma} (1 - e^{i\vec{q}\cdot\vec{a}_{\gamma}})$, where the sum on γ is over all nearest neighbors. It is straightforward to show that $U(\vec{r})$ is nothing but the ‘lattice Coulomb potential’ of a unit negative charge at the origin, with the zero of the potential set at $\vec{r} = \vec{0}$. That is, $U(\vec{r})$ satisfies the ‘lattice Poisson equation’: $\sum_{\gamma} U(\vec{r} - \vec{a}_{\gamma}) - zU(\vec{r}) = -\delta_{\vec{r},\vec{0}}$. For a symmetrical (e.g., square, hexagonal, cubic) lattice, the left-hand side is just the ‘lattice Laplacian’, approaching $\nabla^2 U \times O(a^2)$ where ($a \equiv |\vec{a}_{\gamma}|$).

The quantity Γ in (11) and (12) is then clearly just equal to the potential energy of a neutral (since $\sum_i s_i = 0$) plasma of quantized (since the s_i 's are integers) charges s_i on the lattice. The most relevant $J([s_i])$ is therefore clearly the one that corresponds, in this Coulomb analogy, to the lowest interaction energy. (Note that strictly speaking Γ corresponds to *twice* this energy, because the sum in (12) double counts). Aside from the trivial configuration in which all the $s_i = 0$, the lowest energy configuration is clearly one in which there are two equal and opposite unit magnitude charges on nearest neighbor sites: i.e., $s_0 = +1, \vec{\delta}_0 = \vec{0}, s_1 = -1, \vec{\delta}_1 = \vec{a}_{\gamma}$. As discussed earlier, this corresponds to the nearest-neighbor Josephson coupling in equation (5). Thus, we have established that that coupling is, indeed, the most relevant, as asserted earlier. Furthermore, using simple symmetry arguments, one can show that for a symmetric

lattice (e.g., square, hexagonal, cubic), where all nearest-neighbor sites are equivalent, $U(\vec{a}_\gamma) = -\frac{1}{2z_0}$, which recovers the recursion relation (8) for V . Hence, all other couplings are *irrelevant* for $\alpha \leq \alpha_c = 1/z_0$; as a result they affect neither the floating phase nor the transition between it and the $(D+1)$ -dimensionally coupled phase.

In summary, as long as $\alpha < 1/z_0$, V , and all of the other $J([s_i])$'s are irrelevant. The $(0+1)$ -dimensional systems constitute locally critical power-law phases. Since J 's are also irrelevant in this regime, the local systems are in a floating phase [1]. As soon as V becomes relevant at $\alpha > 1/z_0$, it grows to ∞ . In this limit, this enormous periodic potential obviously traps the field $\Delta\theta$ at the bottom of one of the minima of the periodic potential; and provides a mass for fluctuations about that minimum. Such a mass destroys the logarithmically divergent fluctuations, which give rise to the $\Gamma([s_i])/\alpha$ term in the recursion relation in Eq. 11 for J . Thus, in this limit that recursion relation just becomes $dJ/dl = J$. This in turn makes all J 's relevant simultaneously with the nearest neighbor coupling V , ensuring that there are no further transitions as α is increased. Thus, there is only one global transition with α as the tuning-parameter.

Even though the J 's become relevant as soon as V does, they remain irrelevant far enough on the α -axis for the $(0+1)$ -dimensional transition to happen. The situation is reversed for planes of power-law XY -phases with interlayer couplings, and so Ref. [1] had to introduce extra gradient-couplings among the planes to get floating power-law phases. In the present case, the metal to superconductor quantum phase transition obtained by tuning dissipation in arbitrary spatial dimension is then a true $(0+1)$ -dimensional phase transition. In the floating local critical phase on the metallic side of the transition, the local systems are, however, not decoupled; J 's are irrelevant, but not zero, and affect the physical properties of the metallic phase only perturbatively.

To obtain higher order corrections to the recursion relation for V , Eq. 8, we perform a cumulant expansion of Eq. 7 in second order in V . We find no contribution to Eq. 8 in $D = 1$, while for $D = 2$ triangular lattice there is a contribution at order V^2 , which is

$$\frac{dV}{dl} = V(1 - \frac{1}{z_0\alpha}) + C_1 V^2, \quad (13)$$

where C_1 is a positive constant [20]. This holds for any lattice in any dimension, as long as there are just three sides in a minimum closed loop. For $2D$ square lattice, however, there is no renormalization of V at order V^2 . Indeed, the first correction to Eq. 8 comes only at order V^3 [20] for any lattice where the minimum closed loop has number of sides more than three,

$$\frac{dV}{dl} = V(1 - \frac{1}{z_0\alpha}) + C_2 V^3, \quad (14)$$

where C_2 is another positive constant [20]. It is impor-

tant to emphasize that the line of fixed points implied by Eq. 13 and 14 exist in any dimension, which implies that the upper critical dimension is infinity. Combined with the flow equation for α , $\frac{d\alpha}{dl} = 0$, which remains true in our perturbative treatment at all orders in V , we get the RG flows, the phase diagram, and the exponents quoted earlier.

Not only the existence of a floating phase and its validation in the resistively shunted Josephson junction arrays are exciting possibilities, but the present analysis should find its use in a much broader context: local quantum critical points are widely discussed in heavy fermion systems [21] and are also argued to be crucial to the ubiquitous metallic phases observed in a diverse class of condensed matter systems [22].

This work was supported by the NSF under grants: DMR-0411931, DMR-0132555 and DMR-0132726. We thank S. Kivelson and B. Spivak for discussions, and the Aspen Center for Physics for their hospitality while a portion of this work was being completed.

-
- [1] C. S. O'Hern, T. C. Lubensky, and J. Toner, Phys. Rev. Lett. **83**, 2745 (1999).
 - [2] V. J. Emery *et al.*, Phys. Rev. Lett. **85**, 2160 (2000).
 - [3] C. S. O'Hern, T. C. Lubensky, Phys. Rev. Lett. **80**, 4345 (1998); L. Golubović and M. Golubović, Phys. Rev. Lett. **80**, 4341 (1998).
 - [4] T. Saldit, I. Koltover, J.O. Rädler, and C. R. Safinya, Phys. Rev. Lett. **79**, 2582 (1997); F. Artzner, R. Zantl, G. Rapp, and J.O. Rädler, Phys. Rev. Lett. **81**, 5015 (1998).
 - [5] See, for example, A. M. Goldman, Physica E **18**,1 (2003).
 - [6] S. Chakravarty *et al.*, Phys. Rev. Lett. **56**, 2003 (1986);
 - [7] M. P. A. Fisher, Phys. Rev. B **36**, 1917 (1987).
 - [8] S. Chakravarty *et al.* Phys. Rev. B **37**, 3283 (1988).
 - [9] M. P. A. Fisher, Phys. Rev. Lett. **57**, 585 (1986).
 - [10] M. V. Feigelman and A. I. Larkin, Chem. Phys. **235**, 107 (1998); B. Spivak, A. Zyuzin and M. Hruska, Phys. Rev. B **64**, 132502 (2001). These papers use an effective action different from ours. The description of the floating phase remains unchanged, however, but the line of unstable fixed points in the ordered phase does not exist.
 - [11] Y. Takahide *et al.*, Phys. Rev. Lett. **85**, 1974 (2000); H. Miyazaki *et al.* Phys. Rev. Lett. **89**, 197001 (2002).
 - [12] C. L. Kane and M. P. A. Fisher, Phys. Rev. B **46**, 15233 (1992).
 - [13] A. M. Kadin, K. Epstein, and A. M. Goldman, Phys. Rev. B **27**, 6691-6702 (1983).
 - [14] We exclude "loops" of two bonds made by simply retracing the same bond twice in opposite directions.
 - [15] A. O. Caldeira and A. J. Leggett, Ann. Phys. **149**, 374 (1983).
 - [16] A. Schmid, in *The art of measurement: metrology in fundamental and applied physics*, edited by B. Kramer (VCH Verlagsgesellschaft mbH, Weinheim, 1988)
 - [17] Some of the correlation functions depend on $1/E_0$ singularly; to calculate these we must keep a non-zero $1/E_0$. In this sense $1/E_0$ is a "dangerously irrelevant" variable.

- [18] See, e.g., S. T. Chui and J. D. Weeks, Phys. Rev. B **14**, 4978 (1976).
- [19] J. Toner and D. P. DiVincenzo, Phys. Rev. B **41**, 632 (1990).
- [20] S. Tewari, J. Toner, and S. Chakravarty, in preparation.
- [21] Q. M. Si *et al.*, Nature **413**, 804 (2001).
- [22] A. Kapitulnik *et al.*, Phys. Rev. B **63**, 125322 (2001).